11d Electric-Magnetic Duality and the Dbrane Spectrum

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Abstract

We consider the *gedanken* calculation of the pair correlation function of spatially-separated macroscopic string solitons in strongly coupled type IIA string/M theory, with the macroscopic strings wrapping the eleventh dimension. The supergravity limit of this correlation function with well-separated, *pointlike* macroscopic strings corresponds to having also taken the IIA string coupling constant to zero. Thus, the pointlike limit of the *gedanken* correlation function can be given a precise worldsheet description in the 10D weakly-coupled type IIA string theory, analysed by us in hep-th/0007056 [Nucl. Phys. **B591** (2000) 243]. The requisite type IIA string amplitude is the supersymmetric extension of the worldsheet formulation of an off-shell closed string tree propagator in bosonic string theory, a 1986 analysis due to Cohen, Moore, Nelson, and Polchinski. We point out that the evidence for pointlike sources of the zero-form field strength provided by our worldsheet results clarifies that the electric–magnetic duality in the Dirichlet-brane spectrum of type II string theories is *eleven*-dimensional.

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1 Introduction

The discovery of Dirichlet pbranes in the 10d type II string theories [1], with $-1 \leq p \leq 9$, also raises a puzzle for 10d electric-magnetic duality; let us begin by explaining this puzzle. Recall that Dbranes are domain wall topological defects in an embedding target spacetime that are found to occur naturally among the ground states of type I, and type II, supersymmetric open and closed string theories [1]. In addition to being charged under an antisymmetric (p+2)-form field strength, Dpbranes carry worldvolume Yang-Mills gauge fields. An important distinction from generic solitonic solutions of classical gauge, or supergravity, theories is that string perturbation theory at weak coupling is well-defined in the background of the infinitely thin Dpbrane soliton [4]. Thus, the quantum scattering amplitudes of Yang-Mills gauge theory in the background of a pbrane soliton of *zero width* can be obtained from the low energy gauge theory limit of corresponding perturbative string amplitudes computed in the background of a Dpbrane [3]. This long-unsolved problem in gauged soliton dynamics cannot be addressed by standard field theoretic techniques.

The key property of supersymmetric open and closed string theories that enables the solution of a problem which is surprisingly clumsy to formulate within the semi-classical collective coordinate formalism is the existence of target spacetime duality transformations linking pairs of string vacuum states: $R \to R' = \alpha'/R$, interchanging target spactimes with large, and small, radius. Thus, the existence of vacua with Dirichlet-brane target spacetime geometries can be straightforwardly established by performing a sequence of T-duality transformations on the standard vacuum with the 10D flat spacetime Lorentzian geometry [1]. Moreover, this description is *exact*: it is not necessary to resort to the complicated alternative that describes a Dbrane soliton as composed out of the collective modes of an ensemble of, more fundamental, open and closed strings.

An important step forward in our understanding of the significance of Dbranes is provided by Polchinski's identification of Dbranes as the BPS states of the type II supergravities carrying Ramond-Ramond (R-R) charge [1]. This leap of insight establishes contact between the worldsheet formulation of infinitely thin domain wall pbrane solitons carrying Yang-Mills gauge fields and the zero width limits of a known family of type II classical supergravity soliton solutions [7]. R-R solitons contain worldvolume gauge fields and, for overlapping Dbranes, the gauge group is nonabelian. This implies a noncommutative structure for spacetime at short distances [4], an insight that follows straightforwardly from the worldsheet formulation [1]. Most importantly, as becomes clear with the collective insights assembled together in the landmark work [2], the R-R sector solitons are the precise BPS states known to be of significance in establishing the conjectured strong-weak coupling duality relations linking the different backgrounds of String/M theory.

By application of a sequence of T-duality transformations on the standard ten-dimensional flat spacetime Lorentzian vacuum we can infer, therefore, a spectrum of Dpbranes with p in the range: $-1 \leq p \leq 9$, covering D-instantons thru D9branes [1]. This raises the following puzzle. By a generalization of the Dirac quantization condition for electric and magnetic point charges in four spacetime dimensions, one would expect that the product of the quantum of charge for a Dpbrane, and that of its d-dimensional Poincare dual D(d-p-4)brane, satisfies the relation:

$$\nu_p \nu_{d-4-p} = 2\pi n, \qquad n \in \mathrm{Z} \quad . \tag{1}$$

The spectrum of values for p listed above does not cover all of the expected charges when d=10: we are apparently missing a "D(-2)brane" and a "D(-3)brane", the 10d Poincare duals of the D8brane and D9brane. This is especially puzzling because, in his groundbreaking work [1], Polchinski has shown that the quantum of Dpbrane charge could be computed from first principles using the worldsheet formalism, predicting also that the value of n in the Dirac quantization relation is *unity*. Thus, although we were expecting to find evidence for electric-magnetic duality in the full Dpbrane spectrum we appear, instead, to have found a direct clash with Poincare-Hodge duality in ten target spacetime dimensions.

We will show in this paper that there is a simple resolution to this puzzle, and the evidence in favor of it can be convincingly found in an extension of Polchinski's worldsheet calculation of the Dpbrane tension [1]. In particular, we will argue that the Dirichlet-pbrane spectrum of the type II string theories actually contains *twelve* elements: $-1 \leq p \leq 9$, plus the pointlike sources of zero-form field strength, in full agreement with Poincare-Hodge duality in eleven target spacetime dimensions. The additional Dirichlet-brane is the source for the scalar field strength, F_0, appearing in Roman's massive 10d type IIA supergravity theory [9]; F_0 is the Poincare dual of the 10-form field strength coupling to Polchinski's D8brane [1]. In section 2 of this paper, we will begin by reviewing some of the subtleties in finding evidence for solitonic target spacetime sources for the zero-form field strength, using standard 10d supergravity, and superstring, techniques. In section 3, we formulate our *gedanken* calculation in 11d strongly coupled IIA string/M theory, elucidating the nature of the worldsheet evidence for pointlike sources of the Romans zero-form field strength given by us in [26]. The details of the worldsheet analysis appear in the appendices of the paper. Our conclusions in section 4 focus on the implications of our results for nonperturbative string/M theory.

2 The Magnetic Dual of D8brane Charge

It is helpful to begin by considering the ten-dimensional type IIA, and type IIB, supersymmetry algebras. We remind the reader that Poincare duality in d spacetime dimensions [8, 1, 22] relates a (p+2)–form field strength, F_{p+2}, to its dual field strength, $F_{d-2-p} = *F_{p+2}$, of rank $d-p-2$. A priori, the rank can take any value in the range 0 to d. Supersymmetry places additional restrictions and so, in ten dimensions, for example, we have $p+2 = 0, 2, 4, 6, 8, 10$ in the non-chiral type IIA algebra, and $p+2 = 1, 3, 5, 7, 9$ in the chiral type IIB algebra. Each value of p determines a different central extension of the 10d $N=2$ supersymmetry algebra with 32 supercharges. Thus, in the IIA algebra, we have:

$$\begin{aligned}
\{Q_\alpha, \bar{Q}_\beta\} = & \quad \left[P_\mu + (2\pi\alpha')^{-1} Q^{\mathrm{NS}}_\mu\right] \Gamma^\mu_{\alpha\beta} \\
\{\tilde{Q}_\alpha, \bar{\tilde{Q}}_\beta\} = & \quad \left[P_\mu - (2\pi\alpha')^{-1} Q^{\mathrm{NS}}_\mu\right] \Gamma^\mu_{\alpha\beta} \\
\{Q_\alpha, \bar{\tilde{Q}}_\beta\} = & \quad \sum_{p=0}^{8} \frac{\tau_p}{p!} Q^{\mathrm{R}}_{\mu_1\cdots\mu_p} \left(\Gamma\Gamma^{\mu_1} \cdot \Gamma\Gamma^{\mu_2} \cdots \Gamma\Gamma^{\mu_p}\right)_{\alpha\beta} \quad .
\end{aligned} \tag{2}$$

Here, p takes only even values, and μ runs over all ten target spacetime coordinate labels, the $\{\Gamma^\mu, \Gamma = \Gamma^0\Gamma^1 \cdot \Gamma^9\}$ are the Dirac matrices in the 10d Clifford algebra, and the Q_α, $\tilde{Q}_\alpha$, α=1, $\cdots$, 16

are, respectively, left-moving and right-moving supercharges. The anticommutator of any two chiral supercharges closes on the generator of 10d spacetime translations, and on a central extension in the Neveu-Schwarz (NS) sector: the fundamental string, which couples to the NS sector two-form potential. Nominally, one might expect that the sum on the right-hand-side of the last equation could begin with $p = 0$: it is known that the massive type IIA supergravity action contains both a ten-form field strength [9], as well as its Poincare dual scalar field strength [11]. Could we be missing an additional contribution to the right-hand-side of this equation representing the charge of a Dirichlet (-2)brane?

It turns out that there is no conflict with the ten-dimensional algebra of supercharges given above because the missing Dpbrane of interest will turn out to be of *eleven* dimensional origin: the algebra above contains only ten translation generators, whereas the strongly coupled type IIA string/M theory is eleven dimensional. Notice, also, that the supersymmetry algebra of relevance is that describing the strong coupling limit of Romans' *massive* type IIA supergravity. This is not the same thing as 11d supergravity, the strong coupling limit of the *massless* type IIA supergravity [5, 6, 2], and so we must invoke the supersymmetry algebra of nonperturbative string/M theory itself. Namely, for clear-cut algebraic evidence of sources for the zero-form field strength, we require a framework incorporating both the weak, and strong, coupling limits of the massive type IIA string theory. We will return to this point in the conclusions.

Next, let us move on to consider some aspects of the correspondence between the 10d type IIA superstring theory, and its low energy 10d type IIA supergravity limit. Recall that, for any value of p, the flux quantum, ν_p, differs from the quantum of Dpbrane charge, μ_p, and the physical Dpbrane-tension, τ_p, in its lack of dependence on the closed string coupling, g, and fundamental string tension, $\alpha'^{-1/2}$. As a consequence, it can be calculated unambiguously in *weakly-coupled* perturbative string theory [1]. Moreover, Polchinski showed that the flux quanta of Dirichlet pbranes could be computed from first principles by worldsheet methods [1], satisfying the Dirac quantization condition given above with n equal to unity. Namely, we have the relations:

$$\kappa^2\tau_p^2 \equiv g^2\kappa_{10}^2\tau_p^2 = \kappa_{10}^2\mu_p^2, \qquad \kappa_{10}^2 \equiv \tfrac{1}{2}(2\pi)^7\alpha'^4 \quad . \tag{3}$$

Here, κ is the physical value of the ten-dimensional gravitational coupling. and the dimensionless closed string coupling, $g = e^{\Phi_0}$, where Φ_0 is the vacuum expectation value of the dilaton field. The physical Dbrane tension, τ_p, is the coefficient of the worldvolume action for the Dpbrane [1]. Finally, recall that the flux emanating from a Dirichlet pbrane source is obtained by integrating F_{p+2} over a sphere in $(p+2)$ dimensions that encloses the source:

$$\int_{S_{p+2}} F_{p+2} = 2\kappa_{10}^2\mu_p = \nu_p \quad . \tag{4}$$

The spacetime Lagrangian for the bosonic sector of Romans' 10D massive IIA supergravity theory was obtained in [11] . Note that both the ten-form field strength, as well as its Poincare dual scalar field strength appear explicitly in the spacetime Lagrangian [11, 22]. In the Einstein frame metric, the bosonic part of the massive IIA action takes the simple form [11, 22]:

$$S = \frac{1}{2\kappa^2}\int d^{10}X\sqrt{-G}\left(R_G - \tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{2}|H_3|^2 - \tfrac{1}{2}e^{5\Phi/2}M^2\right) + \frac{1}{2\kappa^2}\int d^{10}X\sqrt{-G}MF_{10} \quad . \tag{5}$$

where M is an auxiliary field to be eliminated by its equation of motion. F_{10} is the non-dynamical ten-form field strength, which can be dualized to a zero-form, or scalar, field strength, $*F_{10}$. This non-dynamical constant field generates a uniform vacuum energy density that permeates the ten-dimensional spacetime, thus behaving like a cosmological constant: $F_{10} = dC_9$, varying with respect to the R-R gauge potential, C_9, gives M = constant, and varying with respect to M gives, $F_{10} = Me^{5\Phi/2}V_{10}$, where V_{10} is the volume of spacetime. Thus, we can identify the dualized scalar field strength with the mass parameter: $*F_{10} = Me^{5\Phi/4}$. Polchinski identified the D8brane as the source for this ten-form field strength in [1], and a solution to the beta function equations with the identifiable spacetime geometry and background fields of the D8brane was subsequently discovered [11, 7]. Can we likewise identify a classical soliton of the 10D massive IIA supergravity theory that can be interpreted as a point source of the zero-form field strength?

Since the worldvolume of a D(-2)brane is "(-1)-dimensional", it is obvious that we cannot look for a suitable solitonic solution to the beta function equations of the massive type IIA theory Ramond-Ramond sector with identifiable worldvolume spacetime geometry, as was done for the remaining members of the Dbrane spectrum [1, 7]. Instead, we will infer the existence of sources for the scalar field strength by calculating the long-range force between two pointlike defects in a solitonic target space geometry. These pointlike defects should not be confused with Dinstantons, a point that will become clear in the next section. Notice that our calculation *is* a precise analog of Polchinski's calculation of the long-range force between generic Dpbranes, because the Dpbranes are always "pointlike" from the perspective of the observor in the bulk target spacetime, for any value of p [1]. It is only the precise power law behavior of the long-range force that identifies the extended worldvolume of the Dpbranes, $R^{-(7-p)}$ for a pair of Dpbranes. Thus, for point sources of the zeroform field strength, we are looking for a R^{-9} fall-off in the long-range force.

It turns out that there exists a fundamental string soliton in the massive 10D type IIA supergravity with the requisite target spacetime geometry that provides a helpful analog for our *gedanken* calculation in strongly coupled type IIA string/M theory in the next section [26]. Soliton solutions to the beta function equations of the 10D massive IIA supergravity in the D0-D8-F1 sector have been exhaustively studied in [13, 12, 29]. Consider a fundamental string terminating at a point within the worldvolume of a D8brane [10]. Generically, this configuration will break 1/4 of the supersymmetries of the type IIA string. Since the Killing spinor is annihilated by the projections of both D8brane and fundamental string, we would infer that the intersection of the worldvolumes of the D8brane and F1 string is a D0brane. This is because the product of either two projection operators gives the third:

$$\hat{\Pi}_{D8} = \frac{1}{2}(1+\Gamma^9), \quad \hat{\Pi}_{F1} = \frac{1}{2}(1+\Gamma^{09}), \quad \hat{\Pi}_{D0} = \frac{1}{2}(1+\Gamma^0\Gamma_{11}) \quad . \tag{6}$$

The observation that the termination of a fundamental string in the worldvolume of a Dpbrane, with all p+1 translation invariances broken, should act like a point source of the magnetic charge dual to the Dpbrane charge, is originally due to Polchinski and Strominger [10]. There exists, not surprisingly, a classical soliton solution of the massive IIA supergravity with precisely these properties among the soliton solutions of the D0-D8-F1 sector of the massive IIA supergravity theory analyzed by Massar and Troost [13], [13], and independently by Janssen, Meessen, and Ortin [12]. We will describe the target spacetime geometry as given in the paper by Massar and Troost [13, 26]. These authors found a new solution to the string beta function equations at leading

order in the α' expansion which describes the following spacetime geometry: a fundamental string extending in the direction, X^9, orthogonal to the worldvolume of an D8brane carrying, in addition, C_1, and B_2, fields. With the notation $z{=}X^9$, $t{=}X^0$, the background fields take the form:

$$\begin{aligned} ds^2 = & \quad -H^{1/8}h^{-13/8}dt^2 + H^{9/8}h^{-5/8}dz^2 + H^{1/8}h^{3/8}((dr)^2 + r^2(d\Omega_7)^2) \\ H(z) = & \quad c + Mz, \quad h(r) = 1 + \frac{Q}{r^6} \\ e^{\Phi} = & \quad H^{-5/4}h^{1/4}, \quad B_{tz} = -h^{-1}, \quad C_z = Hh^{-1} \quad . \end{aligned} \tag{7}$$

The fundamental string extends in the direction perpendicular to the worldvolume of the D8brane, generically breaking 1/4 of the supersymmetries. A nice discussion of this soliton and its extension under $SL(2,Z)$ to a whole multiplet of (p,q) soliton strings, first pointed out by us in [26], has appeared recently in [29].

We will show in the next section that, from the perspective of an observer in the worldvolume of the D8brane, the endpoint of the fundamental string soliton can behave like a point source of Dirichlet (-2)brane charge. The 10D observor can therefore measure the zero string coupling remnant of a calculation that, strictly speaking, belonged in the 11D strongly coupled type IIA string theory. The massive fundamental string soliton of interest to us will wrap the orthogonal spatial coordinate X^{10}, rather than X^9, and its pointlike limit, $R_{10} \to 0$, yields physics that is accessible in the low energy limit of the 10D weakly coupled type IIA string theory. This is the key point that enables a precise calculation of the *tension* of the pointlike source of zeroform field strength using standard type IIA worldsheet techniques [25, 26].

3 Pointlike Sources of Magnetic D(-2)brane Charge

The motivation for our *gedanken* calculation came from an analogous worldsheet calculation in type II string theory. If pointlike sources of the zeroform field strength exist, it would clearly be helpful to find a first-principles calculation of the *tension* of such pointlike sources in the worldsheet formalism, analogous to Polchinski's well-known analysis for the remaining Dpbranes, $-1 \leq p \leq 9$ [1]. It turns out that the answer is to be found in the relatively poorly-explored worldsheet formalism for *macroscopic* loop amplitudes in the perturbative string theories [18, 24, 25], as was pointed out by us in [26].

The worldsheet formulation of macroscopic loop amplitudes, starting from an extension of the standard Polyakov string path integral, was first explored by Cohen, Moore, Nelson, and Polchinski [18]. These authors derived an expression for a covariant off-shell closed string tree propagator in bosonic string theory: the expectation value for a closed bosonic string to propagate from a fixed loop, $\mathcal{C}_i$, in the embedding target spacetime to a different fixed loop, $\mathcal{C}_f$, thru a spatial distance R. Ref. [18] limited their discussion to the case of pointlike boundary loops. In recent works by Chaudhuri, Chen, and Novak, the analysis in [18] has been extended to the case of macroscopic boundary loops in both the bosonic [24], and type II supersymmetric [25, 26], string theories, and including the results for target spacetime backgrounds with an external two-form field strength. In this section, we will begin by reviewing this formalism, explaining why the factorization limit of the

macroscopic loop amplitude in type I$'$ theory yields an expression for the tension. As a byproduct of our analysis, we will also succeed in computing the long range force between a pair of pointlike sources of zeroform field strength in the presence of an external two-form field strength, a result that appears in [26, 25].

It is helpful to clarify precisely where our analysis will differ from Polchinski's computation of the force between two pointlike spacetime events, namely, a pair of Dinstantons. Recall that the Dpbrane tension, $-1 \le p \le 9$, was extracted from a computation of the graviton-dilaton one-point function on the disk lying in the worldvolume of the Dpbrane. A simple trick, exploited in [1], that gives an unambiguously normalized disk one-point function, is to extract the one-point function from the factorization limit of the annulus graph, with boundaries lying in the worldvolume of a pair of parallel, static Dpbranes. For the case of pointlike Dinstantons, the annulus amplitude is computed with all ten embedding target spacetime coordinates obeying the Dirichlet boundary condition [1].

Now consider the anomaly-free and perturbatively renormalizable weakly coupled 10d type IIA string theory in the background with 32 D8branes. One of the two supersymmetries of the IIA string has been broken by the presence of orientifold planes at $X^9{=}0$, and $X^9{=}R_9$, and all 32 D8branes lie in the orientifold plane at the origin. A T-duality transformation maps this background to an analogous background with 32 D9branes in the 10d type IIB string theory. The stack of coincident Dpbranes carries worldvolume Yang-Mills gauge fields with gauge group $SO(32)$. At finite string coupling, an eleventh target space "dimension" emerges, corresponding to the vacuum expectation value of the scalar dilaton field [2]. Consider the following *gedanken* calculation in the strongly coupled IIA string theory:[2] we evaluate the pair correlation function of a pair of spatially separated Wilson loops wrapping the eleventh dimension. The limit of pointlike loops, and large spatial separation, corresponds to taking $R_{10} \rightarrow 0$, and hence we recover a result in *ten-dimensional* type IIA supergravity. Our thought experiment must, therefore, have a precise analog in the factorization limit of a suitable weakly coupled perturbative type IIA string amplitude with the worldsheet topology of an annulus. The relevant computation is as follows: consider the Polyakov path integral summing over worldsurfaces with the topology of an annulus, and with Dirichlet boundary conditions on all ten embedding target space dimensions. We will require further that the boundaries are mapped to a pair of given pointlike loops, $\mathcal{C}_i$, $\mathcal{C}_f$, in the target spacetime. The loops are taken to be spatially separated by a distance R in the X^8 direction, lying within the worldvolume of a stack of 32 coincident D8branes on a single $O8$ plane. This augmented boundary value problem for embedded Riemann surfaces with the topology of an annulus is the precise supersymmetric type IIA analog of a computation carried out in 1986 by Cohen, Moore, Nelson, and Polchinski [18] for the bosonic string theory: the off-shell closed string tree propagator between pointlike loops. An analysis of the type IIA macroscopic loop amplitude appears in a paper by myself and Novak in [25], and in generic background two-form field. The factorization limit of the amplitude with pointlike loops yields the long-range interaction of a pair of supergravity sources, and the result was found by me to be consistent with the tension of a Dirichlet (-2)brane [26].

[2] This is only a *gedanken* calculation at the present time because the strongly coupled IIA string theory is, more precisely, M theory compactified on an $S^1 \times S^1/Z_2$, and we do not know how to calculate in that theory beyond its low energy 11-dimensional supergravity limit.

The detailed derivation of our expression for the macroscopic loop amplitude is given in the Appendix. We perform the sum over worldsheets with the topology of an annulus, and with boundaries mapped to a pair of pointlike loops within the D8brane worldvolume which also carries a constant two-form background.[3] We will orient the parallel loops, $\mathcal{C}_i$, $\mathcal{C}_f$, to lie in a plane perpendicular to their distance of nearest separation, R, along the spatial coordinate X^8. We emphasize that all ten translation invariances of the pointlike loops are forbidden: we have imposed the Dirichlet boundary conditions on all of the X^μ. Such boundary conditions can be implemented from first principles on an extension of the covariant Polyakov string path integral [18, 24, 25]. The Polyakov action contributes a classical piece corresponding to the saddle-point of the quantum string path integral; the saddle-point is determined by the minimum action worldsurface spanning the given loops $\mathcal{C}_i$, $\mathcal{C}_f$. The result for a generic classical solution of the Polyakov action was given by Cohen, Moore, Nelson, and Polchinski in Ref. [18].

The basic methodology for the computation of the macroscopic loop amplitude is reviewed in detail in the Appendix. We emphasize that worldsheet supersymmetry of the type IIB string theory does not introduce any new features in the treatment of the measure for moduli; this was clarified by myself and Novak in [25]. The main difference from Polchinski's analysis of the exchange amplitude for the remaining members of the Dpbrane spectrum, is implementation of the macroscopic loop constraints: we wish to sum over all surfaces with the topology of an annulus, but with boundaries mapped on to a pair of prescribed curves, $\mathcal{C}_i$, $\mathcal{C}_f$ in the embedding target spacetime. This necessitates that all 10 worldsheet scalars in the path integral are Dirichlet: the Dinstanton boundary condition of [1]. But, in addition, one must preserve the full super-Weyl $\times$ superdiffeomorphism invariance in summing over maps from the boundaries of the annulus to a given pair of loops in the target spacetime. This aspect of the Wilson loop boundary value problem was first addressed for pointlike loops in [18], and extended to the case of macroscopic loops in [18, 24, 25]. The result is an additional contribution to the bosonic measure for moduli in the path integral coming from the sum over boundary einbeins.

Our result for the connected sum over worldsurfaces with the topology of an annulus, and with boundaries mapped onto spatially separated macroscopic loops, $\mathcal{C}_i$, $\mathcal{C}_f$, of common length L takes the form [18, 25]:

$$\begin{aligned}\mathcal{A} \quad &= i\left[L^{-1}(4\pi^2\alpha')^{1/2}\right]\int_0^\infty \frac{dt}{2t}\cdot(2t)^{1/2}\cdot e^{-R^2t/2\pi\alpha'}\frac{1}{\eta(it)^8}\times \\ &\left[\left(\frac{\Theta_{00}(0,it)}{\eta(it)}\right)^4-\left(\frac{\Theta_{01}(0,it)}{\eta(it)}\right)^4-\left(\frac{\Theta_{10}(0,it)}{\eta(it)}\right)^4\right]. \end{aligned} \tag{8}$$

This result is derived in pedagogical detail in the Appendix. The only change in the measure for moduli is the additional factor of $(2t)^{1/2}$ contributed by the functional determinant of the bosonic vector Laplacian evaluated on the one-dimensional boundary [18, 24]. Perhaps not surprisingly,

[3]One could, with no loss of generality, refer to these as Wilson loops, as we have done in [24, 25, 26], since the D8brane worldvolume carries nonabelian gauge fields and the endpoint of open strings transforms in the fundamental representation of the Yang-Mills gauge group. In this section, our focus is on pointlike loops and hence on the type IIA supergravity interpretation.

there are no additional contributions in the supersymmetric type II amplitude as was clarified by us in [25].

The pre-factor in square brackets is of interest. A priori, since we have broken translational invariance in all 10 directions of the embedding worldvolume, we expect to see no spacetime volume dependence in the prefactor. If we were only interested in the point-like off-shell closed string propagator, as in [18], the result for the amplitude would be correct without any need for a pre-factor. However, we have *required* that the boundaries of the annulus are mapped to loops in the embedding spacetime of an, a priori, fixed length L. Since a translation of the boundaries in the direction of spacetime parallel to the loops is equivalent to a boundary diffeomorphism, we must divide by the (dimensionless) factor: $L(4\pi^2\alpha')^{-1/2}$. This accounts for the volume dependence in the pre-factor present in our final result. Note that for more complicated loop geometries, the normalization of this expression can take a more complicated form [24]. Further discussion appears in Appendix A.2.

Let us take the factorization limit of the amplitude, expressing the Jacobi theta functions in an expansion in powers of $q{=}e^{-2\pi/t}$. The small t limit is dominated by the lowest-lying closed string modes and the result is:

$$\begin{aligned}\mathcal{A} = \quad & i\left[L^{-1}(4\pi^2\alpha')^{1/2}\right]\int_0^\infty dt\cdot(2t)^{-1/2}\cdot t^4\cdot q^{-1}\left(1+8q+O(q^2)\right)e^{-R^2t/2\pi\alpha'} \\ \rightarrow \quad & iL^{-1}8\cdot 2^{-8}(4\pi^2\alpha')^5\pi^{-9/2}\Gamma\left(\frac{9}{2}\right)|R|^{-9} \quad .\end{aligned} \tag{9}$$

Repeating the steps in Polchinski's calculation of the Dbrane tension [1, 22, 28], we infer the existence of a Dirichlet (-2)brane in the massive type II string theory with tension:

$$\tau_{-2}^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^5 \quad . \tag{10}$$

One of the interesting properties of the BPS states in the type II brane spectrum is their isomorphic ultraviolet-infrared behaviour: the force law between Dpbranes takes identical form at short and long distances. The pointlike source of zeroform field strength is no different in this respect, as already noted in [26]. Let us quote the result for the full short-distance asymptotic expansion of the force between these sources in the presence of a constant electromagnetic background field. We comment that this formal expression was derived in [24, 25], prior to a full appreciation of its physical interpretation as the short distance force law for point sources of the IIA scalar field strength described by us in [26]:

$$\begin{aligned}V(r,\alpha) = \quad & -L^{-1}(4\pi^2\alpha')^{1/2}\frac{\tanh(\pi\alpha)}{\pi\alpha}\frac{1}{r} \\ & \times\left[\sum_{k=1}^\infty C_k z^{2k}\gamma\left(2k+\tfrac{1}{2},1/z\right)+\sum_{k=1}^\infty\sum_{m=1}^\infty C_{k,m}z^{2(k+m)}\gamma\left(2(k+m)+\tfrac{1}{2},1/z\right)\right],\end{aligned} \tag{11}$$

where the $\gamma(2n+\frac{1}{2},1/z)$ are the incomplete gamma functions. The asymptotic expansion has been developed in terms of the natural choice of dimensionless variable: $z = r_{\rm min}^2/r^2$. Here, $r_{\rm min}^2 = 2\pi\alpha'(\pi\alpha) = 4\pi^2\alpha'|\alpha'\bar{F}_2|$, is the minimum distance accessible in open string theory in the presence of

a constant background electromagnetic field, $|\bar{F}_2|$ [21]. The coefficients in the asymptotic expansion are given by [24]:

$$
\begin{aligned}
C_k = &\quad \frac{4(-1)^k(2^{2k}-4)}{(2k!)}\pi^{2k} \\
C_{k,m} = &\quad \frac{8(-1)^k(2^{2m-1}-1)}{(2k!)(2m!)}|B_{2m}|(2^{2k}-4)\pi^{2(k+m)} \quad ,
\end{aligned}
\tag{12}
$$

where the B_{2m} are the Bernoulli numbers. Notice that the coefficient of the k=1 term in both sums vanishes, so that the leading contribution to the short distance potential behaves as $-\alpha^4/r^9$, precisely as expected for point sources of the zeroform field strength [22]:

$$
\begin{aligned}
V(r,\alpha) = &\quad -L^{-1}(4\pi^2\alpha')^{1/2}\int_0^{\alpha^{-1}} dt e^{-r^2t/2\pi\alpha'}(2t)^{1/2}\frac{\tanh\pi\alpha}{\mathrm{Sin}\pi\alpha t}\left[12+4\mathrm{Cos}(2\pi\alpha t)-16\mathrm{Cos}(\pi\alpha t)\right] \\
= &\quad -L^{-1}\frac{\alpha^4}{r^9}2^6\pi^{11/2}\Gamma\left(\frac{9}{2}\right)\alpha'^{9/2}+O(\alpha^6) \quad .
\end{aligned}
\tag{13}
$$

The range of integration has been taken to span the first period of the sine function, $0 \le \alpha t \le 1$, as discussed in [22, 24]. This result is a concrete indication of pointlike sources of the supergravity zeroform field strength, an identification first made by us in [26].

4 Conclusions

We have found a satisfying resolution to the puzzle raised in the Introduction that appears to soldifies the case in favor of an *eleven*-dimensional electric-magnetic duality in String/M Theory. Poincare-Hodge duality in 11 dimensions would imply a Dirichlet pbrane spectrum with twelve elements, with p taking values in the range: $-2 \le p \le 9$. This is precisely the spectrum of Dpbrane charges for which we have found evidence in the worldsheet formalism of the perturbative type I and type II supersymmetric string theories.

In particular, we have demonstrated explicitly that Polchinski's worldsheet calculation of the Dpbrane tension, with p taking values in the range $-1 \le p \le 9$ in the type II string theories [1], admits a unique extension that is natural both from the perspective of the boundary value problem for Riemann surfaces, as well as from the target spacetime perspective of generalized electric-magnetic duality in the R-R soliton spectrum. We have evaluated the pair correlation function of a pair of spatially separated Wilson loops wrapping the eleventh dimension. The limit of pointlike loops, and large spatial separation, corresponds to taking $R_{10} \to 0$, and it must have a precise analog in the factorization limit of a suitable weakly coupled perturbative type IIA string amplitude with the worldsheet topology of an annulus. We have shown that the requisite worldsheet calculation yields the tension of a pointlike source of the zeroform field strength in String/M theory.

The massive soliton strings in the type IIA D0-F1-D8 system [12, 13] had the target spacetime geometry of a solitonic F1 string with D0brane charge spread smoothly along the length of an

F1 string extending in the direction orthogonal to the worldvolume of an D8brane. It is natural to suspect that such a type I$'$ soliton string can be uplifted to a soliton background for M theory compactified on $S^1 \times S^1/Z_2$ with the following target space geometry: a fundamental string extending along the interval X^{10} between orientifold planes in eleven dimensions, terminating in the D9branes on the "walls" bounding the interval. It would be most interesting if the arguments in this paper, and the supergravity results in [12, 13], could be extended to show that the endpoint of the F1 string behaves like a source of Dirichlet (-2)brane charge in eleven dimensions. The zero form, and eleven form, field strengths are Poincare duals in eleven dimensions, and we would like to interpret the absence of a D(-3)brane in the Dpbrane spectrum of the type II string theories as unambiguous indication of *the 11d nature of electric magnetic duality in the Dbrane spectrum of the type II string theories.*

So what does this tell us more generally about Poincare-Hodge duality in String/M theory? In [30], we have explained why Poincare duality in eleven dimensions, consistent with the appearance of precisely 12 elements in the Dirichlet pbrane spectrum of the type II string theories, $-2 \leq p \leq 9$, could be perfectly compatible with a fundamental theory of emergent spacetime based upon degrees of freedom that are *zero-dimensional* U(N) matrices [30]. The emergence of dualities in the large N limits of this theory has to do with the choice of Duality group, determining the precise form of the matrix Lagrangian [30]. Notice that as many as eleven noncompact coordinates can indeed emerge in the different large N limits of the matrix Lagrangians described in [30]. The full significance of electric-magnetic duality in nonperturbative string/M theory remains to be discovered.[4]

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Appendix: Macroscopic Loop Amplitudes

For completeness, we are presenting this derivation in full pedagogical detail. The macroscopic loop amplitude sums over worldsurfaces with two boundaries. Thus, there is only one contributing worldsheet topology at leading order in string perturbation theory, even in the unoriented open and closed string theories.

A.1 Review of the Dpbrane Exchange Amplitude

We begin by reviewing the computation of the one-loop amplitude of bosonic open and closed string theory from world-surfaces with the topology of a cylinder following Polchinski's analogous first principles path integral derivation for the torus in [17], and including the extensions to p+1 Dirichlet boundaries [1], with $-1 \leq p \leq 9$, and an external two form field strength [20, 19, 23]. From the perspective of the closed string channel, this amplitude represents the tree-level propagation of a

[4] An expanded discussion of electric-magnetic dualities from this perspective appears in [31].

single closed string, exchange between a spatially-separated pair of Dpbranes. A crucial observation is as follows: although the Dpbrane vacuum corresponds to a spontaneous breaking of translation invariance in the bulk $25-p$ dimensional space orthogonal to the pair of Dpbranes, notice that spacetime translational invariance is preserved within the p+1-dimensional worldvolume of each Dpbrane. At the end of this sub-section, we can specialize to the case of Dinstantons with all 26 embedding target space scalars satisfying Dirichlet boundary conditions.

One further clarification is required to distinguish results in the presence, or absence, of Dirichlet p-branes. In the absence of Dpbranes, the boundary of the worldsheet can lie in all 26 dimensions, and we impose Neumann boundary conditions on all d=26 scalars. This gives the traditional open and closed string theory, whose supersymmetric generalization is the type IB string theory. T-dualizing $25-p$ embedding coordinates gives the open and closed string theory in the background geometry of a pair of Dpbranes [1]. Its supersymmetric generalization is the type I$'$ string theory, when p is even, and the type IB string theory in generic Dpbrane background, when p is odd [1]. The Dpbranes define the hypersurfaces bounding the compact bulk spacetime, which is ($25-p$)-dimensional. Since the bulk spacetime has edges, these $25-p$ embedding coordinates are Dirichlet worldsheet scalars. It is conventional to align the Dpbranes so that the distance of nearest separation, R, corresponds to one of the Dirichlet coordinates, call it X^{25}.

In the presence of a pair of Dpbranes, the classical worldsheet action contributes a background term given by the Polyakov action for a string of length R stretched between the Dpbranes [1, 22, 28]:

$$S_{\rm cl}[G,g] = \tfrac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{g} g^{ab}\, G^{25,25}(X)\partial_a X^{25}\partial_b X^{25} = \frac{1}{2\pi\alpha'}R^2 t \quad , \tag{14}$$

where the second equality holds in the critical dimension on open world-surfaces of vanishing Euler number. The background dependence, $e^{-S_{\rm cl}[G,g]}$, appears in the sum over connected open Riemann surfaces of any topology, orientable or nonorientable. Notice that the background action is determined both by the fiducial worldsheet metric, g_{ab}, *and* by the bulk spacetime metric, $G_{\mu\nu}[X]$. Notice also that the boundary of an open worldsheet is now required to lie within the worldvolume of the Dpbrane, although the worldsheet itself is embedded in all 26 spacetime dimensions.

The metric on the generic annulus can be parameterized by a single real worldsheet modulus, t, and it takes the form:

$$ds^2 = e^{\phi}((d\sigma^1)^2 + 4t^2(d\sigma^2)^2), \quad \sqrt{g} = e^{\phi}2t, \quad 0 \le t \le \infty \quad , \tag{15}$$

with worldsheet coordinates, σ^a, a=1, 2, parameterizing a square domain of unit length. $2t$ is the physical length of either boundary of the annulus as measured in the two-dimensional field theory on the worldsheet. The differential operator mapping worldsheet vectors, $\delta\sigma^a$, to symmetric traceless tensors, usually denoted $(P_1\delta\sigma)_{ab}$, has only one zero mode on the annulus. This is the constant diffeomorphism in the direction tangential to the boundary: $\delta\sigma_0^2$. Likewise, the analysis of the zero modes of the scalar Laplacian must take into account the Dpbrane geometry: the $p+1$ noncompact embedding coordinates satisfying Neumann boundary conditions are treated exactly as in the case of the torus. The $25-p$ Dirichlet coordinates lack a zero mode. Thus:

$$1 = \int d\delta X e^{-\frac{1}{2}|\delta X|^2} = \prod_{\mu=0}^{p}\int d\delta\bar{x} e^{-\frac{1}{2}(\delta\bar{x}^\mu)^2\int d^2\sigma\sqrt{g}} \int d\delta X' e^{-\frac{1}{2}|\delta X'|^2}$$

$$= \quad (2\pi)^{(p+1)/2}\left(\int d^2\sigma\sqrt{g}\right)^{-(p+1)/2}\int d\delta X' e^{-|\delta X'|^2/2} \quad . \tag{16}$$

The analysis of the diffeomorphism and Weyl invariant measure for moduli follows precisely as for the torus, differing only in the final result for the Jacobian [28]. We have:

$$\begin{aligned} 1 = \quad & \int dg dX e^{-\frac{1}{2}|\delta g|^2-\frac{1}{2}|\delta X|^2} \\ = \quad & (\det Q_{22})^{-1/2}\,\frac{\left(\int d^2\sigma\sqrt{g}\right)^{1/2+(p+1)/2}}{(2\pi)^{(p+1)/2}}\,(\det{}'\mathcal{M})^{1/2}\int (d\phi d\delta\sigma)' dt dX' e^{-\frac{1}{2}|\delta g|^2-\frac{1}{2}|\delta X|^2}, \end{aligned} \tag{17}$$

where $(\det Q_{22}) = 2t$ in the critical dimension, cancelling the factor of $2t$ arising from the normalization of the integral over the single real modulus. As shown in [28], $\mathcal{M}$ takes the form:

$$(\det{}'\mathcal{M})^{1/2} = (\det{}'\ 2\ \Delta_d^c)^{1/2}\left(\frac{1}{2t}\right) = \frac{1}{2}(2t)^{-1}\det{}'\Delta = \tfrac{1}{2}(2t)^{-1}\prod_{n_2=-\infty}^{\infty}\ \prod_{n_1=-\infty}^{\infty\prime}\omega_{n_2,n_1} \quad . \tag{18}$$

The Laplacian acting on free scalars on an annulus with boundary length $2t$ takes the form, $\Delta{=}(2t)^{-2}\partial_2^2{+}\partial_1^2$, with eigenspectrum:

$$\omega_{n_2,n_1} = \frac{\pi^2}{t^2}(n_2^2+n_1^2t^2), \quad \Psi_{n_2,n_1} = \frac{1}{\sqrt{2t}}e^{2\pi i n_2\sigma^2}\mathrm{Sin}(\pi n_1\sigma^1) \quad , \tag{19}$$

where the subscripts take values in the range $-\infty{\le}n_2{\le}\infty$, and $n_1{\ge}0$ for a Neumann scalar, or $n_1{\ge}1$ for a Dirichlet scalar.

In the case of a background electromagnetic field, $\mathcal{F}_{p-1,p}$, it is convenient to complexify the corresponding pair of scalars: $Z{=}X^p{+}iX^{p-1}$ [19, 20, 28]. They satisfy the following twisted boundary conditions:

$$\partial_1 \mathrm{Re}Z = \quad \mathrm{Im}Z = 0 \quad \sigma^1 = 0 \tag{20}$$

$$\partial_1 \mathrm{Re}e^{-i\phi}Z = \quad \mathrm{Im}e^{-i\phi}Z = 0 \quad \sigma^1 = 1 \tag{21}$$

Expanding in a complete set of orthonormal eigenfunctions gives:

$$Z = \sum_{n_2,n_1} z_{n_2,n_1}\Psi_{n_2,n_1} = \frac{1}{\sqrt{2t}}e^{2\pi i n_2\sigma^2}\mathrm{Sin}\pi(n_1+\alpha)\sigma^1 \quad , \tag{22}$$

where $\pi\alpha{=}\phi$, and $\pi{-}\phi$, respectively [28], and subscripts take values in the range $-\infty{\le}n_2{\le}\infty$, $n_1{\ge}0$. The twisted complex scalar has a discrete eigenvalue spectrum on the annulus given by:

$$\omega_{n_2,n_1}(\alpha) = \frac{\pi^2}{t^2}(n_2^2+(n_1+\alpha)^2t^2) \quad . \tag{23}$$

Thus, the connected sum over worldsurfaces with the topology of an annulus embedded in the target spacetime geometry of a pair of parallel Dpbranes separated by a distance R in the direction X^9, and in the absence of a magnetic field, takes the form [28]:

$$W_{\mathrm{ann}} = \prod_{\mu=0}^{p} L^\mu \int_0^\infty \frac{dt}{2t}(8\pi^2\alpha' t)^{-(p+1)/2}\eta(it)^{-24}e^{-R^2t/2\pi\alpha'} \quad . \tag{24}$$

In the presence of a worldvolume electromagnetic field, $\mathcal{F}_{p-1,p}$, the scalars X^{p-1}, X^p, are complexified. Substituting the result for the eigenspectrum of the twisted complex scalar gives:

$$W_{\rm ann}(\alpha) = \prod_{\mu=0}^{p-2} L^{\mu} \int_0^{\infty} \frac{dt}{2t} (8\pi^2\alpha' t)^{-(p-1)/2} \eta(it)^{-22} e^{-R^2 t/2\pi\alpha'} \frac{e^{\pi t\alpha^2}\eta(it)}{i\Theta_{11}(it\alpha, it)} \quad , \tag{25}$$

with $\alpha{=}i\phi/\pi$ and $q{=}e^{-2\pi t}$.

A.2 The Extension to Macroscopic Boundary Loops

We move on to the extension to macroscopic boundary loops, first explored by Cohen, Moore, Nelson, and Polchinski [18]. So far, we have performed the Weyl $\times$ Diffeormorphism invariant sum over all worldsurfaces with the topology of an annulus, and with fully Dirichlet boundaries. We now wish to perform a gauge invariant sum over all maps of the boundaries of the annulus to a pair of fixed curves, $\mathcal{C}_i$, $\mathcal{C}_f$ in the embedding target spacetime. The main modification is to the sum over metrics on the boundary: it would be too strong to impose Dirichlet boundary condition on the deformations of the worldsheet metric. Instead, we can parametrize the generic map from worldsheet boundary to target space loop in terms of a boundary einbein [18]. What remains is to perform the sum over the set of *gauge-inequivalent* maps of worldsheet boundary to target space loop.

We should pause to point out that it is this last step which was not completed in [18], except for the case of pointlike loops, where it is trivial. In [24], we realized that, for large loop length, the gauge invariant sum over arbitrary deformations of the einbein simply amounts to adapting the sum over gauge-inequivalent deformations of the bulk worldsheet metric to those of the boundary einbein [15, 17]. For more topologically nontrivial Wilson loop configurations, the normalization of the macroscopic loop amplitude can indeed contain additional numerical factors arising from the discrete component of the sum over inequivalent maps. But the worldsheet modulus dependence in the measure of the modular integral will be unchanged from the simplest case of single-winding circular Wilson loops. The modulus dependence in the measure of the string path integral is of crucial physical significance, since it determines the precise power law behavior of the long-range potentials obtained in the low energy field theory limits of the string amplitude. See the comments below Eq. (30), and in Footnote 6.

For simplicity, it is convenient to align the macroscopic loops, $\mathcal{C}_i$, $\mathcal{C}_f$, which we choose to have the common length L, such that their distance of nearest separation, R, is parallel to a spatial coordinate, call it X^{25}. As in the previous sub-section, the Polyakov action contributes a classical piece corresponding to the saddle-point of the quantum path integral: the saddle-point is determined by the minimum action worldsurface spanning the given loops $\mathcal{C}_i$, $\mathcal{C}_f$. The result for a generic classical solution of the Polyakov action was derived in [18]. For coaxial circular loops in a flat spacetime geometry, the result is identical to that for a spatially separated pair of generic Dpbranes in flat spacetime, namely Eq. (14). Notice, in particular, that there is no L dependence in the saddle-point action as a consequence of the Dirichlet boundary condition on all 26 scalars. As in the case of Dinstanton boundary conditions considered in [1], we evaluate the determinant of the scalar Laplacian for all 26 embedding coordinates with the Dirichlet boundary condition. Notice that there is no contribution from coordinate zero modes, since all of the X^{μ} are Dirichlet. Thus, the

usual box-regularized spacetime volume dependence originating in the Neumann sector is absent, precisely as in the vacuum of a pair of Dinstantons. The analog of Eq. (16) reads:

$$1 = \int d\delta X e^{-\frac{1}{2}|\delta X|^2} = \int d\delta X' e^{-|\delta X'|^2/2} \quad . \tag{26}$$

The crucial difference in the path integral computation when the boundaries of the annulus are mapped to *macroscopic* loops in embedding spacetime has to do with the implementation of boundary reparameterization invariance: we must include in the path integral a sum over all possible maps of the worldsheet boundary to the loops $\mathcal{C}_i$, $\mathcal{C}_f$ [18]. Notice that the analysis of reparametrization invariance in the bulk of the worldsheet is unaltered. As a consequence, the conditions for Weyl invariance, and for the crucial decoupling of the Liouville mode, are unchanged.

Let us proceed with the analysis of the measure following the steps in [15, 17]. The differential operator mapping worldsheet vectors, $\delta\sigma^a$, to symmetric traceless tensors, usually denoted $(P_1\delta\sigma)_{ab}$, has only one zero mode on the annulus. This is the constant diffeomorphism in the direction tangential to the boundary: $\delta\sigma_0^2$. The analysis of the diffeomorphism and Weyl invariant measure for moduli follows precisely as for the annulus.[5]. The only difference is an additional contribution from the vector Laplacian, accounting for diffeomorphisms of the metric which are nontrivial on the boundary [18]. The analog of Eq. (17) now takes the form:

$$\begin{aligned} 1 = & \int de \int dg dX e^{-\frac{1}{2}|\delta g|^2 - \frac{1}{2}|\delta X|^2 - \frac{1}{2}|\delta e|^2} \\ = & \left(\det Q_{22}\right)^{-1/2} \int d^2\sigma \sqrt{g} \left(\det{}' \mathcal{J}\right)^{1/2} \left(\det{}' \mathcal{M}\right)^{1/2} \int (d\phi d\delta\sigma)' dt dX' e^{-\frac{1}{2}|\delta g|^2 - \frac{1}{2}|\delta X|^2 - \frac{1}{2}|\delta e|^2}, \end{aligned} \tag{27}$$

where $(\det Q_{22}) = 2t$ in the critical dimension, canceling the factor of $2t$ arising from the normalization of the integral over the single real modulus. The functional determinant of the vector Laplacian acting in the worldsheet bulk takes the form:

$$\left(\det{}' \mathcal{M}\right)^{1/2} = \left(\det{}' \, 2 \, \Delta_d^c\right)^{1/2} \left(\frac{1}{2t}\right) = \frac{1}{2}(2t)^{-1} \det{}' \Delta = \tfrac{1}{2}(2t)^{-1} \prod_{n_2=-\infty}^{\infty} \prod_{n_1=-\infty}^{\infty\prime} \omega_{n_2,n_1} \quad , \tag{28}$$

and the infinite product is computed as in appendix A.3. The functional determinant of the operator $\mathcal{J}$ can likewise be expressed in terms of the functional determinant of the Laplacian acting on free scalars on the one-dimensional boundary, parameterized here by σ^2 [18]. Thus, for boundary length $2t$, we have $\Delta_b = (2t)^{-2}\partial_2^2$, with eigenspectrum:

$$\omega_{n_2} = \frac{\pi^2}{t^2} n_2^2, \quad \Psi_{n_2} = \frac{1}{\sqrt{2t}} e^{2\pi i n_2 \sigma^2} \quad , \tag{29}$$

where the subscripts take values in the range $-\infty \leq n_2 \leq \infty$. The result can be read off as a special case of the expressions in appendix A.3.

[5]A pedagogical derivation can be found in the electronic review [28]

Thus, the connected sum over worldsurfaces with the topology of an annulus with boundaries mapped onto spatially separated macroscopic loops, $\mathcal{C}_i$, $\mathcal{C}_f$, of common length L takes the form [18, 24]:

$$\mathcal{A}_{i,f} = \left[L^{-1}(4\pi^2\alpha')^{1/2}\right]\int_0^\infty \frac{dt}{2t}\cdot(2t)^{1/2}\cdot\eta(it)^{-24}e^{-R^2t/2\pi\alpha'} \quad . \tag{30}$$

The only change in the measure for moduli is the additional factor of $(2t)^{1/2}$ contributed by the functional determinant of $\mathcal{J}$. The pre-factor in square brackets is of interest; recall that there is no spacetime volume dependence in this amplitude since we have broken translational invariance in all 26 directions of the embedding spacetime. If we were only interested in the point-like off-shell closed string propagator, as in [18], the result as derived is correct without any need for a pre-factor. However, we have *required* that the boundaries of the annulus are mapped to loops in the embedding spacetime of an, a priori, fixed length L. Since a translation of the boundaries in the direction of spacetime parallel to the loops is equivalent to a boundary diffeomorphism, we must divide by the (dimensionless) factor: $L(4\pi^2\alpha')^{-1/2}$. This accounts for the pre-factor present in our final result. Note that for more complicated loop geometries, including the possibility of Wilson loops with multiple windings, or even cusps and corners [24], the numerical pre-factor in this expression can take a far more complicated form.

A3 The Macroscopic Loop Amplitude in Type II String Theories

The derivation of the gauge invariant measure for moduli given for the bosonic string can be easily extended to the case of the supersymmetric unoriented open and closed string theories. in generic type II string backgrounds with Dbranes [1, 22, 23, 25, 26]. We begin with the contribution from worldsurfaces with the topology of an annulus in the presence of a background electromagnetic field, and in the background spacetime geometry of a pair of Dpbranes separated by a distance R. This expression was derived in [24]:

$$\begin{aligned} W_{ann-I}(\alpha) = \quad & \prod_{\mu=0}^{p-2}\int_0^\infty \frac{dt}{2t}(8\pi^2\alpha' t)^{-(p-1)/2}\eta(it)^{-6}e^{-R^2t/2\pi\alpha'}\frac{e^{\pi t\alpha^2}\eta(it)}{i\Theta_{11}(it\alpha, it)} \\ & \times \prod_{n_2=0}^{\infty}\prod_{n_1=-\infty}^{\infty\prime}\left(\det{}_{\mathrm{ann-I}}\Delta_{n_2+\frac{1}{2},n_1+\frac{1}{2}+\alpha}\right)^1\left(\det{}_{\mathrm{ann-I}}\Delta_{n_2+\frac{1}{2},n_1+\frac{1}{2}}\right)^3, \end{aligned} \tag{31}$$

where we have included the contribution from worldsheet bosonic fields derived in the previous section.

Let us understand the eigenvalue spectrum of the worldsheet fermions in more detail. Recall that the functional determinant of the two-dimensional Dirac operator acting on a pair of Majorana Weyl fermions satisfying twisted boundary conditions is equivalent, by Bose-Fermi equivalence, to the functional determinant of the scalar Laplacian *raised to the inverse power*. This provides the correct statistics. In addition, we have the constraint of world-sheet supersymmetry. This requires that the four complexified Weyl fermions satisfy identical boundary conditions in each sector of the theory in the σ^1 direction. For a complex Weyl fermion satisfying the boundary condition:

$$\begin{aligned} \psi(1,\sigma^2) = \quad & -e^{\pi i a}\psi(0,\sigma^2) \\ \psi(\sigma^1,1) = \quad & -e^{\pi i b}\psi(\sigma^1,0) \quad , \end{aligned} \tag{32}$$

the Bose-Fermi equivalent scalar eigenspace takes the form:

$$\Psi_{n_2+\frac{1}{2},n_1+\frac{1}{2}(1+a)} = \frac{1}{\sqrt{2t}} e^{2\pi i(n^2+\frac{1}{2}(1\pm b))\sigma^2} \mathrm{Cos}\pi(n_1 + \tfrac{1}{2}(1\pm a)) \quad , \tag{33}$$

where we sum over $-\infty \leq n_2 \leq \infty$, $n_1 \geq 0$. Notice that the unrotated oscillators are, respectively, half-integer or integer moded as expected for the scalar equivalent of antiperiodic or periodic worldsheet fermions. Finally, we must sum over periodic and antiperiodic sectors, namely, with a, b equal to 0, 1. As reviewed in the appendix, weighting the (a, b) sector of the path integral by the factor $e^{\pi iab}$ gives the following result for the fermionic partition function [22]:

$$\begin{aligned} Z_b^a(\alpha, q) = &\quad q^{\frac{1}{2}a^2-\frac{1}{24}} e^{\pi iab} \prod_{m=1}^{\infty} \left[(1 + e^{\pi ib} q^{m-\frac{1}{2}(1-a)+\alpha})(1 + e^{-\pi ib} q^{m-\frac{1}{2}(1+a)-\alpha}) \right] \\ \equiv &\quad \frac{1}{e^{\pi t\alpha^2}\eta(it)} \Theta_{ab}(\alpha it, it) \quad . \end{aligned} \tag{34}$$

We have included a possible rotation by α or $1-\alpha$ as in the previous section. This applies for the Weyl fermion partnering the twisted complex worldsheet scalar. Substituting in the path integral, and summing over a, b=0, 1, for all fermions, and over α and $1-\alpha$ for the Weyl fermion partnering the twisted complex scalar, gives the result:

$$\begin{aligned} W_{\mathrm{ann-I}} &= \prod_{\mu=0}^{p-2} L^\mu \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-(p-1)/2} \eta(it)^{-6} e^{-R^2 t/2\pi\alpha'} \left[\frac{e^{\pi t\alpha^2}\eta(it)}{\Theta_{11}(it\alpha, it)} \right] \\ &\times \left[\frac{\Theta_{00}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{00}(0, it)}{\eta(it)} \right)^3 - \frac{\Theta_{01}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{01}(0, it)}{\eta(it)} \right)^3 - \frac{\Theta_{10}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{10}(0, it)}{\eta(it)} \right)^3 \right] \end{aligned} \tag{35}$$

where we have used the fact that $\Theta_{11}(0, it)$ equals zero. The analogous expression for the macroscopic loop amplitude can be straightforwardly written down, since the only changes are in the analysis of boundary deformations of the worldsheet metric. More details can be found in [25]. The result takes the form [25, 26]:

$$\begin{aligned} \mathcal{A} &= i \left[L^{-1} (4\pi^2\alpha')^{1/2} \right] \int_0^\infty \frac{dt}{2t} \cdot (2t)^{1/2} \cdot e^{-R^2 t/2\pi\alpha'} \left[\frac{e^{\pi t\alpha^2}\eta(it)}{\Theta_{11}(it\alpha, it)} \right] \\ &\times \left[\frac{\Theta_{00}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{00}(0, it)}{\eta(it)} \right)^3 - \frac{\Theta_{01}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{01}(0, it)}{\eta(it)} \right)^3 - \frac{\Theta_{10}(it\alpha, it)}{e^{\pi t\alpha^2}\eta(it)} \left(\frac{\Theta_{10}(0, it)}{\eta(it)} \right)^3 \right] \end{aligned} \tag{36}$$

A.4 Zeta-Regularized Eigenvalue Spectrum of the Twisted Scalar

The regularization of a divergent sum over the discrete eigenvalue spectrum of a self-adjoint differential operator by the zeta function method can always be carried out in closed form when

the eigenvalues are known explicitly [16]. This is the case for all of the infinite sums encountered in one-loop superstring amplitudes, even in the presence of generic background fields [17, 22].[6]

The eigenspectrum of the scalar Laplacian on a surface with boundary includes a dependence on an electromagnetic background, reflected as a twist in the boundary conditions satisfied by the scalar. As an illustration, let us work out the functional determinant of the scalar Laplacian for worldsheets with the topology of an annulus with generic twist α. We begin with a discussion of the eigenspectrum. In the case of the free Neumann scalars, we must introduce an infrared regulator mass for the zero mode, as in the case of the torus [17]. The functional determinant of the Laplacian can be written in the form:

$$\ln \det{}'\Delta = \lim_{m\to 0} \sum_{n_1=0}^{\infty} \sum_{n_2=-\infty}^{\infty} \log \left[\frac{\pi^2}{t^2}(n_2^2 + n_1^2 t^2 + m^2) \right] - \log \left(\frac{\pi^2 m^2}{t^2} \right) \quad . \tag{37}$$

The first term in Eq. (37) is a special case of the infinite sum with generic twist, α, and the zeta-regulated result can be obtained by setting α=0 in the generic calculation which will be derived below. The second term in Eq. (37) yields the result:

$$+ \lim_{m\to 0} \lim_{s\to 0} \frac{d}{ds} [\frac{4\pi^2 m^2}{4t^2}]^{-s} = 2 \log 2t - \lim_{m\to 0} -2 \log (2\pi m) \quad . \tag{38}$$

This contributes the correct power of $2t$ to the measure of the path integral for a free Neumann scalar.[7] For the Dirichlet scalar, we must remember to drop the n_1=0 modes from the double sum above since the sine eigenfunction vanishes for all values of σ^1, not only at the boundary. Thus, the n_1 summation begins from n_1=1. The n_1=n_2=0 term has been included in the infinite sum by introducing an infrared regulator mass, m, for the zero mode. We will take the limit $m \to 0$ at the end of the calculation.

We begin by expressing the first term in Eq. (37) in the equivalent form:

$$S_{\rm ann} = - \lim_{s,m\to 0} \frac{d}{ds} \left\{ (\frac{\pi^2}{t^2})^{-s} \sum_{n_1=0}^{\infty} \sum_{n_2=-\infty}^{\infty} \left[n_2^2 + n_1^2 t^2 + m^2 \right]^{-s} \right\} \quad . \tag{39}$$

Notice that the infinite sums are manifestly convergent for Re $s > 1$. The required $s \to 0$ limit can be obtained by analytic continuation in the variable s. The analogous step for the second term in Eq. (37) yields the relation:

$$+ \lim_{m\to 0} \lim_{s\to 0} \frac{d}{ds} [\frac{\pi^2 m^2}{t^2}]^{-s} = 2 \log t - \lim_{m\to 0} 2 \log (\pi m) \quad . \tag{40}$$

[6] A pedagogical review of Polchinski's calculation of the zeta-regularized functional determinant of the scalar Laplacian on the torus [17] can be found in the electronic review [28], which also contains the analogous derivations for the one-loop open and unoriented string graphs, as well as the supersymmetric extensions, and in the presence of background two-form fields. The two-form background field dependence in generic one-loop scattering amplitudes of the open bosonic string theory was first derived in a paper with Novak [27]. We are only including calculations pertinent to the macroscopic loop amplitude in this paper.

[7] The authors of [18] were careless on this one point in their analysis. Their result, which appears in Eq. (4.5) of Ref. [18], contains an extraneous factor of $t^{-26/2}$ in the measure. This would be appropriate for 26 Neumann scalars, but is clearly absent in an all-Dirichlet bosonic string amplitude [1, 22]. See the comment below Eq. (36).

The finite term in this expression contributes the overall factor of t^2 to the result for the one-loop vacuum amplitude.

The infinite summation over n_2 is carried out using a Sommerfeld-Watson transform as in [17]. We invoke the Residue Theorem in giving the following contour integral representation of the infinite sum as follows:

$$\sum_{n=-\infty}^{\infty} \left[n^2+x^2\right]^{-s} = \oint_{\sum_n \mathcal{C}_n} \frac{dz}{2\pi i} \pi \cot(\pi z) \left(z^2+x^2\right)^{-s} \quad , \tag{41}$$

where $\mathcal{C}_n$ is a small circle enclosing the pole at $z{=}n$ in the counterclockwise sense. The contours may be deformed without encountering any new singularities into the pair of straight line contours, $\mathcal{C}_\pm$, where the line $\mathcal{C}_+$ runs from $\infty{+}i\epsilon$ to $-\infty{+}i\epsilon$, connecting smoothly to the line $\mathcal{C}_-$, which runs from $-\infty{-}i\epsilon$ to $\infty{-}i\epsilon$.

Alternatively, we can choose to close the contours, $\mathcal{C}_\pm$, respectively, in the upper, or lower, half-planes along the circle of infinite radius. Note that the integrand has additional isolated poles in the complex plane at the points $z{=}\pm ix$. We will make the following substitution in the integrand:

$$\begin{aligned} \frac{\mathrm{Cot}(\pi z)}{i} &= \frac{2e^{i\pi z}}{e^{i\pi z}-e^{-i\pi z}} - 1 \quad , \quad \mathrm{Im}\ z>0 \\ \frac{\mathrm{Cot}(\pi z)}{i} &= \frac{-2e^{-i\pi z}}{e^{i\pi z}-e^{-i\pi z}} + 1 \quad , \quad \mathrm{Im}\ z<0 \end{aligned} \tag{42}$$

when the contour is to be closed, respectively, in the upper, or lower, half-plane. This ensures that the integrand is convergent at all points within the enclosed region other than the isolated poles. Thus, we obtain the following alternative contour integral representation of the infinite sum over n_2, setting $x^2{=}n_1^2t^2{+}m^2$:

$$\begin{aligned} \sum_{n_2=-\infty}^{\infty} \left[n_2^2+x^2\right]^{-s} = & \oint_{\mathcal{C}_+} dz \left[\frac{e^{i\pi z}}{e^{i\pi z}-e^{-i\pi z}} - \tfrac{1}{2}\right] \left(z^2+x^2\right)^{-s} \\ & + \oint_{\mathcal{C}_-} dz \left[-\frac{e^{-i\pi z}}{e^{i\pi z}-e^{-i\pi z}} + \tfrac{1}{2}\right] \left[z^2+x^2\right]^{-s} \quad . \end{aligned} \tag{43}$$

Note that the contours are required to avoid the branch cuts which run, respectively, from $+ix$ to $+i\infty$, and from $-ix$ to $-i\infty$.

Now consider the case of the twisted Neumann scalar. There is no need to introduce an infrared regulator in the presence of a magnetic field since there are no zero modes in the eigenvalue spectrum. Thus, the analysis of the infinite eigensum is similar to that for a Dirichlet scalar, other than the incorporation of twist. We begin with:

$$S_{\mathrm{ann}} = -\lim_{s\to 0} \frac{d}{ds} [\frac{\pi^2}{t^2}]^{-s} \sum_{n_2=-\infty}^{\infty\prime} \sum_{n_1=1}^{\infty} \left[n_2^2+(n_1+\alpha)^2t^2)\right]^{-s} \quad , \tag{44}$$

and identical statements can be made about its convergence properties as in the zero external field case. The n_2 summation in S_{ann} is carried out using a contour integral representation with

$x{=}(n_1{+}\alpha)t$. The n_1 summation can be recognized as the Riemann zeta function with two arguments,

$$\sum_{n_1=0}^{\infty}(n_1+\alpha)^{-2s+1}t^{-2s+1}\equiv\zeta(2s-1,\alpha)t^{-2s+1} \quad . \tag{45}$$

Taking the s-derivative followed by the $s{=}0$ limit gives,

$$\begin{aligned}\lim_{s\to 0}\frac{d}{ds} &\quad \zeta(2s-1,\alpha)t^{-2s+1}B(\tfrac{1}{2},s-\tfrac{1}{2})\\ = &\quad \lim_{s\to 0}\frac{d}{ds}\zeta(2s-1,\alpha)t^{-2s+1}\frac{\sin(\pi s)}{\sqrt{\pi}}\Gamma(1-s)\Gamma(s-\tfrac{1}{2})=-2\pi t\zeta(-1,\alpha) \quad .\end{aligned} \tag{46}$$

Substituting the relation $\zeta(-n,q){=}{-}B'_{n+2}(q)/(n+1)(n+2)$, and combining the contributions for $q{=}\alpha$, and $1{-}\alpha$, gives:

$$\frac{\pi t}{3}\left[B'_3(1-\alpha)+B'_3(\alpha))\right]=2\pi t\left[\alpha^2+\frac{1}{6}-\alpha\right] \quad , \tag{47}$$

where we have used $B'_n(q){=}nB_{n-1}(q)$, and $B_2(q){=}q^2{-}q{+}\frac{1}{6}$.

Next, we tackle the non-constant pieces from the square brackets in the analog of Eq. (43). It is helpful to take the s-derivative and the $s\to 0$ limit prior to performing the contour integral. We begin with the contour integral in the upper half-plane:

$$\begin{aligned}I_2(x,s)= &\quad \int_{\mathcal{C}_+}dz\frac{e^{i\pi z}}{e^{i\pi z}-e^{-i\pi z}}\left(z^2+x^2\right)^{-s}\\ = &\quad -2\sin(\pi s)\int_x^{\infty}dy\frac{e^{-\pi y}}{e^{\pi y}-e^{-\pi y}}\left(y^2-x^2\right)^{-s} \quad .\end{aligned} \tag{48}$$

Taking the derivative with respect to s, and setting $s{=}0$, gives:

$$\begin{aligned}\frac{d}{ds}I_2(x,s)|_{s=0}= &\quad -2\pi\int_x^{\infty}dy\frac{e^{-\pi y}}{e^{\pi y}-e^{-\pi y}}\\ = &\quad -\log\left(1+e^{-2\pi x}\right) \quad .\end{aligned} \tag{49}$$

The $\mathcal{C}_-$ integral gives an identical contribution as before. Combining the contributions to $S_{\rm ann}$ from terms with $\alpha{=}\phi/\pi$, and $1{-}\alpha$, respectively, gives the following result in the $m{\to}0$ limit:

$$S_{\rm ann}=2\pi t(\alpha^2+\frac{1}{6}-\alpha)-2\sum_{n_1=0}^{\infty}\log\left[(1+e^{-2\pi(n_1+\alpha)t})(1+e^{-2\pi(n_1+1-\alpha)t})\right] \quad , \tag{50}$$

The result for the functional determinant of the Laplacian acting on a twisted complex scalar takes the form:

$$\left[\prod_{\pm}\prod_{n_1=0}^{\infty}\prod_{n_2=-\infty}^{\infty\prime}\omega_{n_2,n_1}\right]^{-1}=q^{\frac{1}{2}(\alpha^2+\frac{1}{6}-\alpha)}(1-q^{\alpha})^{-1}\prod_{n_1=1}^{\infty}\left[(1-q^{n_1-\alpha})(1-q^{n_1+\alpha})\right]^{-1} \quad , \tag{51}$$

where $q{=}e^{-2\pi t}$. The result can be expressed in terms of the Jacobi theta function as follows:

$$\frac{q^{\frac{1}{2}\alpha^2+\frac{1}{24}-\frac{1}{8}}}{-2i\mathrm{Sin}(\pi t\alpha/2)}\prod_{n_1=1}^{\infty}\Big[(1-q^{n_1-\alpha})(1-q^{n_1+\alpha})\Big]^{-1}=-i\frac{e^{\pi t\alpha^2}\eta(it)}{\Theta_{11}(it\alpha,it)}\quad , \tag{52}$$

with $\alpha{=}\phi/\pi$.

Setting $\alpha{=}0$ in this expression, and combining with the result in Eq. (38), gives the functional determinant of the Laplacian acting on a free Neumann scalar:

$$\left(\prod_{n_2=-\infty}^{\infty\prime}\prod_{n_1=0}^{\infty}\omega_{n_1n_2}\right)^{-1/2}=\left(\frac{1}{2t}\right)q^{-\frac{1}{24}}\prod_{n_1=1}^{\infty}(1-q^{n_1})^{-1}=\frac{1}{2t}\left[\eta(it)\right]^{-1}\quad , \tag{53}$$

where $q{=}e^{-2\pi t}$. The expression for the Dirichlet determninant is identical except for the absence of the overall factor of $1/2t$.

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